Z(5th Sm.)-Mathematics-H/DSE-B-1.1, DSE-B-1.2 & DSE-B-1.3/CBCS

2023

MATHEMATICS — HONOURS

Paper : DSE-B-1.1, DSE-B-1.2 and DSE-B-1.3

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Paper : DSE-B-1.1

(Discrete Mathematics)

Full Marks : 65

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any. (**1 mark** for correct option and **1 mark** for justification.) 2×10

(a) The remainder when the sum $4! + 5! + 6! + ... + 50!$ is divided by 4 is

(i) 1 (ii) 2

(iii) 3 (iv) 0.

(b) If n is odd, then n^2-1 is divisible by

(i) 3 (ii) 8

(iii) 12 (iv) 17.

(c) The highest power of a prime 5 in 213! is

(i) 50 (ii) 51

(iii) 52 (iv) 53.

(d) The maximum number of edges in a connected simple graph with n vertices is

(i) $\frac{n(n+1)}{2}$ (ii) $n-1$

(iii) $\frac{n(n-1)}{2}$ (iv) n^2-n.

(e) The number of edges in a forest with n vertices and k components is

(i) $n+k$ (ii) $n-k$

(iii) nk (iv) $n(n-k)$.

(f) A tree has five vertices of degree 2, five vertices of degree 3 and four vertices of degree 4. How many vertices of degree 1 does it have?

(i) 10 (ii) 11

(iii) 12 (iv) 13.

(g) The order of 5 modulo 17 is

(i) 9 (ii) 11

(iii) 15 (iv) 16.

(h) What is remainder when 16! is divided by 19?

(i) 5 (ii) 7

(iii) 9 (iv) 11.

(i) Which one of the following pair of integers is not congruent modulo 7?

(i) 1, 15 (ii) −1, 8

(iii) 0, 42 (iv) −9, 5.

(j) The value of $\phi(720)$ is

(i) 64 (ii) 96

(iii) 192 (iv) 384.

Unit - I

Answer *any five* questions. 5×5

2. Prove that every connected graph has a spanning tree.

3. Prove that every chain $(<, \leq)$ is a distributive lattice.

4. 11 numbers are chosen from $\{1, 2, 3, .., 20\}$. Prove that one of them will be a multiple of the other.

5. Find the numbers of edge disjoint Hamiltonian cycles in a complete graph k_n, where n is odd and ≥ 3.

6. Prove that a graph is bipartite if and only if it contains no odd cycles.

7. Show that a non-directed graph has an open Eulerian trail if and only if it is connected and has two vertices of odd degree.

8. Let G be a simple graph with n vertices and m edges, where m is at least 3. If $m \geq \frac{1}{2}(n-1)(n-2)+2$, prove that G is Hamiltonian.

9. Show that every simple connected planar graph G must have a vertex of degree ≤ 5.

Unit - II

Answer *any four* questions. 5×4

10. Use Fermat's theorem to prove that $1^{p-1} + 2^{p-1} + 3^{p-1} + \ldots + (p-1)^{p-1} \equiv (-1) \bmod p$, when p is an odd prime.

11. The sum of two positive integers is 100. If one is divided by 7 the remainder is 1, and if the other is divided by 9 the remainder is 7. Find the numbers.

12. Given that $\gcd(a, 4) = 2$ and $\gcd(b, 4) = 2$, prove that $\gcd(a + b, 4) = 2$.

13. If m is an odd prime and a be an integer such that $(a, m) = 1$, then show that $\left(\frac{a}{m}\right) \equiv a^{\frac{m-1}{2}} \pmod m$.

14. If n is a positive integer such that $(n-1)! \equiv -1 \pmod n$, then show that n is prime.

15. If p is an odd prime, then show that any divisor of the Mersenne number $M_p = 2^p - 1$ is of the form $2kp + 1$, where k is a positive integer.

16. Let p be an odd prime and a an integer not divisible by p. Then, prove that the congruence $x^2 \equiv a \pmod p$ has either no solutions of exactly two incongruent solutions modulo p.

Paper : DSE-B-1.2

(Linear Programming and Game Theory)

Full Marks : 65

1. Answer *all* questions with proper explanation / justification (**one** mark for correct answer and **one** mark for justification) : 2×10

(a) Which of the following option determines the extreme point(s) of the set $S = \left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1, x \ge 0, y \ge 0\right\}$?

(i) (0, 0), (a, 0), (0, b)

(ii) All points on the elliptical boundary

(iii) (0, 0), (a, 0), (0, b) and all points on the elliptical boundary in the 1st quadrant

(iv) None of the above.

(b) The union of two convex sets

(i) may or may not be a convex set

(ii) must be a convex set

(iii) must not be a convex set

(iv) none of the above.

(c) The solution (1, $\frac{1}{2}$, 0, 0, 0) of the equations

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

is

(i) a basic solution

(ii) not a basic solution

(iii) a basic feasible solution

(iv) a degenerate solution.

(d) Consider two sets $X = \{(x_1, x_2) : x_1 + x_2 \le 2, 2x_1 + 2x_2 \ge 8, x_1, x_2 \ge 0\}$ and $Y = \{(x_1, x_2) : x_1^2 + x_2^2 \le 4\}$, then

(i) X is convex set but Y is not convex set

(ii) X is not convex set, but Y is a convex set

(iii) Both X and Y are convex sets

(iv) None of X and Y are convex sets.

(e) If any of the constraints in the primal problem be a perfect equality, then the corresponding dual variable is

(i) unrestricted in sign (ii) positive

(iii) negative (iv) zero.

(f) In a Simplex method if the optimality condition be satisfied at any iteration and if the basis containing one or more artificial vectors with corresponding artificial variable at zero level, then the solution obtained is

(i) optimal

(ii) no feasible

(iii) feasible but not optimal

(iv) feasible but not basic.

(g) A degenerate Basic feasible solution in a balanced Transportation problem with m origins and n destinations will consist of

(i) at least $(m + n - 1)$ positive variables

(ii) at most $mn - (m + n - 1)$ positive variables

(iii) at most $(m + n - 1)$ positive variables

(iv) at most $(m + n - 2)$ positive variables.

(h) If either of the primal or dual admits unbounded solution, then the other has

(i) one feasible solution

(ii) two feasible solutions

(iii) no feasible solution

(iv) infinite number of feasible solutions.

(i) In an assignment problem with m jobs and m machines, the number of basic variables at zero level in a basic feasible solution is

(i) m (ii) $m - 1$

(iii) $m + 1$ (iv) $m - 2$.

(j) In a game with 2×2 pay-off matrix

a	b
c	d

where $a < d < b < c$

(i) a is the saddle point (ii) b is the saddle point

(iii) d is the saddle point (iv) there is no saddle point.

Unit - I

2. Answer *any two* questions :

(a) An agricultural firm has 180 tons of Nitrogen fertilizer, 50 tons of Phosphate and 220 tons of Potash. It will be able to sell 3:3:4 mixture of these substances at a profit of ₹ 15 per ton and 2:4:2 mixture at a profit of ₹ 12 per ton respectively. Formulate an L.P.P. to how many tons of these two mixtures should be produced to obtain the maximum. 5

(b) Starting from the feasible solution (1, 2, 4) obtain a basic feasible solution of the system

$$2x_1 + 3x_2 - x_3 = 4$$
$$3x_1 - x_2 + x_3 = 5$$
$$x_1, x_2, x_3 \geq 0.$$
5

(c) Prove that a basic feasible solution to a LPP corresponds to an extreme point of the convex set of feasible solutions. 5

(d) Define convex set. Prove that the set $X = \{(x, y)/x^2 + y^2 \leq 4\}$ is a convex set. 1+4

Unit - II

3. Answer *any one* question :

(a) (i) Solve the following L.P.P. by simplex method.

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0.$

(ii) Prove that the objective function of a Linear Programming Problem assumes its optimal value at an extreme point of convex set of feasible solutions. 6+4

(b) (i) Solve the following L.P.P. by two phase method :

Maximize $Z = 2x_1 + x_2 + x_3$

subject to $4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 + 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0.$

(ii) Use Big-M method to solve the L.P.P. :

Maximize $Z = x_1 + 5x_2$

subject to $3x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \geq 3$

$x_1, x_2 \geq 0.$ 6+4

Unit - III

4. Answer *any one* question :

(a) (i) Solve the following L.P.P. by solving its dual problem :

Maximize $Z = 5x_1 + 4x_2$

subject to $3x_1 + 4x_2 \leq 24$

$3x_1 + 2x_2 \leq 18$

$x_2 \leq 5$

$x_1, x_2 \geq 0.$

(ii) Prove that the dual of the dual of a L.P.P. is the primal. 7+3

(b) (i) A feasible solution x^* to the primal problem is optimal if and only if there exist a feasible solution v^* to the dual problem such that $cx^* = b^T v^*$.

(ii) Formulate the dual of the L.P.P. given below :

Minimize $Z = 4x_1 + 5x_2 - 3x_3$

subject to $x_1 + x_2 + x_3 = 22$

$3x_1 + 5x_2 - 2x_3 \leq 65$

$x_1 + 7x_2 + 4x_3 \geq 120$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign. 6+4

Unit - IV

5. Answer *any three* questions :

(a) Using VAM solve the following transportation problem and find the optimal solution : 5

	D_1	D_2	D_3	D_4	$a_i \downarrow$
O_1	19	30	50	10	7
O_2	70	50	40	60	9
O_3	40	8	70	20	18
$b_j \rightarrow$	5	8	7	14	

(b) Solve the following travelling salesman problem with the following cost matrix $(C_{ij})_{4\times4}$, where C_{ij} is the cost of travelling from city i to city j. 5

To

From	1	2	3	4
1	∞	15	30	4
2	6	∞	4	1
3	10	15	∞	16
4	7	18	13	∞

(c) Find the optimal assignments to find the minimum cost for the assignment problem with the profit matrix given below : 5

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

(d) Reduce the following pay-off matrix to 2×2 matrix by dominance property and thus solve the following Game Problem : 3+2

Player B

Player A	B_1	B_2	B_3	B_4
A_1	4	2	3	2
A_2	−2	4	6	4
A_3	2	1	3	5

(e) Using graphical method, solve the following rectangular game with the following pay-off matrix : 5

Player B

Player A		B_1	B_2
	A_1	2	4
	A_2	2	3
	A_3	3	2
	A_4	–2	6

Please Turn Over

Paper : DSE-B-1.3

(Boolean Algebra and Automata Theory)

Full Marks : 65

1. All are multiple choice questions with single correct option. Students are required to opt the correct option and justify the correct option in their own words as far as practicable. One mark for correct option and one mark for justification. There is no negative marking. (1+1)×10

(a) Any tree contains at least k number of pendant vertices, where k is

(i) 2 (ii) 1

(iii) 3 (iv) 4.

(b) How many strings of length less than 4, contains the language described by the regular expression $(x+y)^*y(a+ab)^*$?

(i) 7 (ii) 10

(iii) 12 (iv) 11.

(c) CFG for a^+

(i) $S \rightarrow aS \mid a \mid \wedge$ (ii) $S \rightarrow aS \mid b$

(iii) $S \rightarrow aS \mid a$ (iv) None of these.

(d) In the lattice $(S, \leq)$, where $S = \{1, 2, 4, 5, 8, 9\}$ and '$\leq$' denotes usual less than or equality relation. Then value of $(2 \vee (2 \wedge 8)) \vee 4$ is

(i) 2 (ii) 4

(iii) 8 (iv) 9.

(e) Which one of the following is the reflexive-transitive closure of the relation $\{(1, 2), (2, 3)\}$?

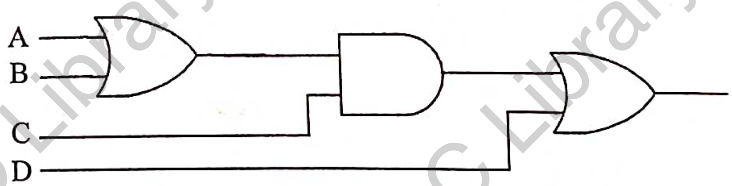
(i) $\{(1, 2), (2, 3), (1, 3)\}$

(ii) $\{(1, 2), (2, 3), (1, 3), (3,1)\}$

(iii) $\{(1, 1), (2, 2), (3, 3), (1, 3), (1, 2), (2, 3)\}$

(iv) $\{(1, 1), (2, 2), (3, 3), (1, 3)\}$.

(f) Which one of the following is the Boolean expression for the logic circuit shown below?



(i) CA+CB+CD (ii) $C(A+B)\bar{D}$

(iii) C(A+B)+D (iv) CA+CB+D.

(g) The set of all strings over $\{a, b\}$ of even length is represented by the regular expression

(i) $(ab + aa + bb + ba)^*$ (ii) $(a + b)^* (a^* + b)^*$

(iii) $(aa + bb)^*$ (iv) $(ab + ba)^*$.

(h) The regular grammar generating $\{a^n : n \geq 1\}$ is

(i) $(\{S\}, \{a\}, \{S \to aS\}, S)$ (ii) $(\{S\}, \{a\}, \{S \to SS, S \to a\})$

(iii) $(\{S\}, \{a\}, \{S \to aS\}, S)$ (iv) $(\{S\}, \{a\}, \{S \to aS, S \to a\}, S)$.

(i) In a deterministic push-down automata, $|\delta(q, a, z)|$ is

(i) equal to 1 (ii) less than or equal to 1

(iii) greater than 1 (iv) greater than or equal to 1.

(j) For the standard Turing Machine $M = (Q, \Sigma, \Gamma, \delta, q_0, b, F)$, which one of the following is true?

(i) $\Sigma = \Gamma$ (ii) $\Gamma \subseteq \Sigma$

(iii) $\Sigma \subseteq \Gamma$ (iv) Σ is a proper subset of Γ.

Unit - I

Answer *any one* question.

2. What is a lattice homomorphism? Give an example to show that the order relations are preserved under lattice homomorphism. 2+2

3. Let D_{20} denote the set of all positive divisors of 20.

(a) Show that $(D_{20}, \leq)$ is a poset, where $a \leq b$ if and only if a divides b.

(b) Is it a lattice? 3+1

Unit - II

Answer *any one* question.

4. Draw the Hasse diagram of the poset $B = (X, \leq)$, where $A = \{1,2,3,4\}$ and $X = P(A)$ and the poset relation is containment. 4

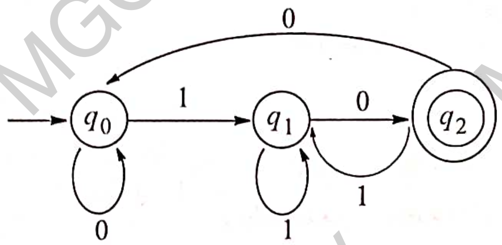
5. For any Boolean Algebra B, prove that $(a + b)(b + c)(c + a) = ab + bc + ca$, for $a, b, c \in B$. 4

Unit - III

Answer *any one* question.

6. Let $G = (\{S, C\}, \{a, b\}, P, S)$ be a grammar, where P consists of $S \to aCa$, $C \to aCa \mid b$. Find the language $L(G)$ generated by G. 4

7. Let M be a DFA whose transition diagram is



(a) Write the transition table for this DFA.

(b) Which of the strings 111010, 0100 and 1110 are accepted by M ? 2+2

Unit - IV

Answer *any two* questions.

8. (a) Prove that the language is not regular : $L = \{a^n b^n : n \geq 1\}$.

(b) Let L be a regular language. Then prove that there exists a regular expression r such that $L = L(r)$.

(c) State one application of pumping lemma. 3+3+1

9. (a) Is there any difference between a finite automation and a finite state machine? Explain.

(b) Construct an NFA that accepts the regular expression 0*(010)*(00+11). Then convert the NFA into DFA. 2+5

10. Let G be a context free grammar given by $S \to AB$, $A \to BS|b$, $B \to SA|a$.

Convert G into Greibach Normal Form (GNF). 7

Unit - V

Answer *any two* questions.

11. Construct a pushdown automata (PDA) that reads the same language as the grammar $\Gamma = (N, \Sigma, S, P)$ defined by $N = \{S, A, B\}$, $\Sigma = \{a, b, c\}$ and the set of productions P given by

$$S \to AB \qquad A \to abaA \qquad A \to \lambda \qquad B \to Bcacc \qquad B \to \lambda$$ 6

12. Find a Turing Machine (not necessarily deterministic) that accepts the context-free language $\{a^n b^n : n = 1, 2, \ldots\}$. 6

13. Convert the grammar with following production rules to Chomsky Normal Form :

$$P = \{S \to ASB \mid \Lambda, A \to aAS \mid a, B \to SbS \mid A \mid bb\}.$$ 6

Unit - VI

Answer *any one* question.

14. (a) Give the transition functions δ (i.e., specify the domain and ranges) of a DFA, NFA, PDA, Turing Machine and non-deterministic Turing Machine.

    (b) Prove that every multi-tape Turing Machine has an equivalent single-tape Turing Machine. 3+4

15. (a) Prove that the recursiveness problem of type 0 grammar is unsolvable.

    (b) Give an example of a language that is not recursive but recursively enumerable. 4+3